Relational Model and Algebra

Introduction to Databases

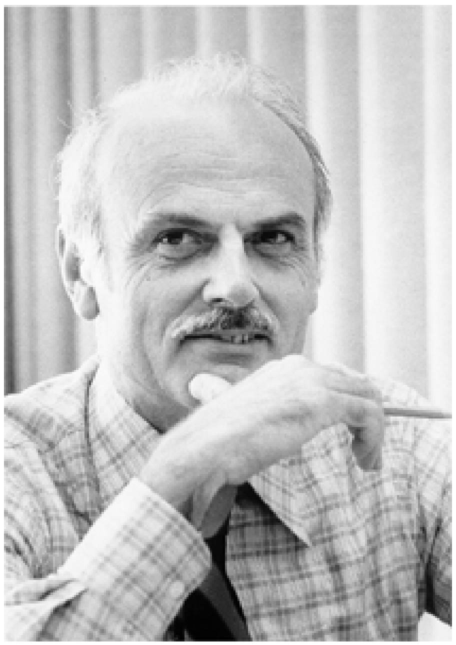
CompSci 316 Fall 2014

DUKE
COMPUTER SCIENCE

Announcements (Thu. Aug. 28)

- Registration
  - As a courtesy to others, please add/drop ASAP
  - Tonight: permission #'s will be emailed to 18 on the wait list
  - Monday evening: another round of permission #'s
  - If you are not on the official wait list, check `http://www.cs.duke.edu/courses/fall14/compsci316/duke-only/more-wait-list.txt`
- UTAs and office hours to be announced soon
- Next week
  - Brett will run the class (I will be away at a conference)
  - Tuesday: lab to help with setup, VM, RA—bring laptop!
  - Thursday: relational database design
- Homework #1 assigned; due in ~2 weeks
  - Sign up for Gradiance and Piazza
  - Wait for our email to start setting up VM (and signing up for Amazon if needed)

Edgar F. Codd (1923-2003)



- Pilot in the Royal Air Force in WW2
- Inventor of the relational model and algebra while at IBM
- Turing Award, 1981

http://en.wikipedia.org/wiki/File:Edgar_F_Codd.jpg

Relational data model

- A database is a collection of relations (or tables)
- Each relation has a set of attributes (or columns)
- Each attribute has a name and a domain (or type)
  - Set-valued attributes are not allowed
- Each relation contains a set of tuples (or rows)
  - Each tuple has a value for each attribute of the relation
  - Duplicate tuples are not allowed
    - Two tuples are duplicates if they agree on all attributes

☞Simplicity is a virtue!

Example

User

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…

Group

gid	*name*
abc	Book Club
gov	Student Government
dps	Dead Putting Society
…	…

Member

uid	*gid*
142	dps
123	gov
857	abc
857	gov
456	abc
456	gov
…	…

Ordering of rows doesn't matter
(even though output is
always in some order)

Schema vs. instance

- Schema (metadata)
  - Specifies how the logical structure of data
  - Is defined at setup time
  - Rarely changes
- Instance
  - Represents the data content
  - Changes rapidly, but always conforms to the schema

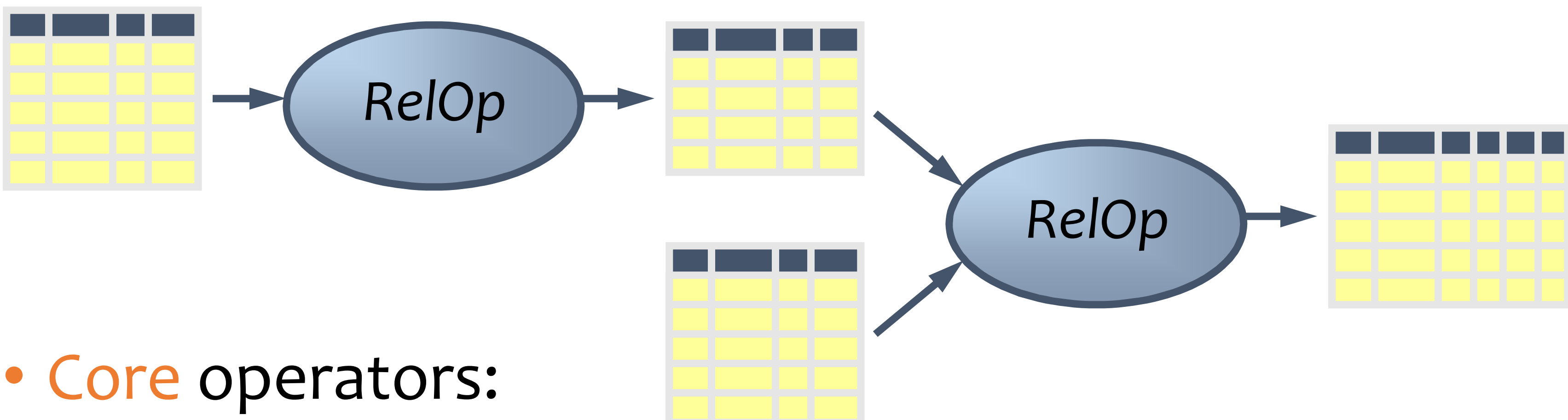
☞Compare to type vs. objects of type in a programming language

Example

- Schema
  - *User* (*uid* int, *name* string, *age* int, *pop* float)
  - *Group* (*gid* string, *name* string)
  - *Member* (*uid* int, *gid* string)
- Instance
  - *User*: {⟨`142`, `Bart`, `10`, `0.9`⟩, ⟨`857`, `Milhouse`, `10`, `0.2`⟩, ...}
  - *Group*: {⟨`abc`, `Book Club`⟩, ⟨`gov`, `Student Government`⟩, ...}
  - *Member*: {⟨`142`, `dps`⟩, ⟨`123`, `gov`⟩, ...}

Relational algebra

A language for querying relational data based on "operators"



- Core operators:
  - Selection, projection, cross product, union, difference, and renaming
- Additional, derived operators:
  - Join, natural join, intersection, etc.
- Compose operators to make complex queries

Selection

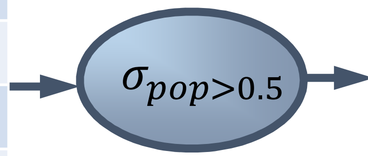
- Input: a table R
- Notation: $\sigma_p R$
  - p is called a selection condition (or predicate)
- Purpose: filter rows according to some criteria
- Output: same columns as R, but only rows or R that satisfy p

Selection example

- Users with popularity higher than 0.5

$$\sigma_{pop>0.5} User$$

uid	name	age	pop
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…



uid	name	age	pop
142	Bart	10	0.9
[redacted]			
857	Lisa	8	0.7
[redacted]			
…	…	…	…

More on selection

- Selection condition can include any column of R, constants, comparison ($=$, $\leq$, etc.) and Boolean connectives (∧: and, ∨: or, ¬: not)
  - Example: users with popularity at least 0.9 and age under 10 or above 12

$$\sigma_{pop \geq 0.9 \wedge (age < 10 \vee age > 12)} \; User$$

- You must be able to evaluate the condition over a single row of the input table!
  - Example: the most popular user

$$\sigma_{pop \geq every\ pop\ in\ User} \; User$$

WRONG!

Projection

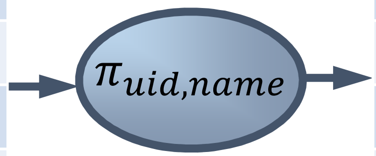
- Input: a table R
- Notation: $\pi_L R$
  - L is a list of columns in R
- Purpose: output chosen columns
- Output: same rows, but only the columns in L

Projection example

- IDs and names of all users

$$\pi_{uid,name}\ User$$

uid	name	age	pop
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…



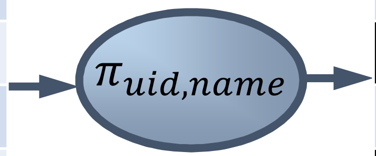
uid	name
142	Bart
123	Milhouse
857	Lisa
456	Ralph
…	…

More on projection

- Duplicate output rows are removed (by definition)
  - Example: user ages

$$\pi_{age}\ User$$

uid	*name*	*age*	*pop*
142	Bart	10	0.9
123	Milhouse	10	0.2
857	Lisa	8	0.7
456	Ralph	8	0.3
…	…	…	…



age
10
[redacted]
8
[redacted]
…

Cross product

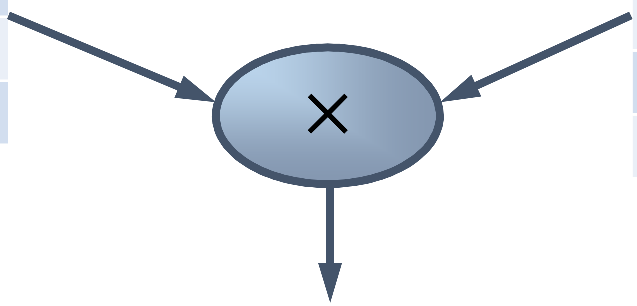
- Input: two tables R and S
- Natation: $R \times S$
- Purpose: pairs rows from two tables
- Output: for each row r in R and each s in S, output a row rs (concatenation of r and s)

Cross product example

$$User \times Member$$

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	gid
123	gov
857	abc
857	gov
…	…



uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

A note a column ordering

- Ordering of columns is unimportant as far as contents are concerned

uid	*name*	*age*	*pop*	*uid*	*gid*
123	Milhouse	10	0.2	123	gov
123	Milhouse	10	0.2	857	abc
123	Milhouse	10	0.2	857	gov
857	Lisa	8	0.7	123	gov
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

=

uid	*gid*	*uid*	*name*	*age*	*pop*
123	gov	123	Milhouse	10	0.2
857	abc	123	Milhouse	10	0.2
857	gov	123	Milhouse	10	0.2
123	gov	857	Lisa	8	0.7
857	abc	857	Lisa	8	0.7
857	gov	857	Lisa	8	0.7
…	…	…	…	…	…

- So cross product is commutative, i.e., for any R and S, $R \times S = S \times R$ (up to the ordering of columns)

Derived operator: join

(A.k.a. "theta-join")

- Input: two tables R and S
- Notation: $R \bowtie_p S$
  - p is called a join condition (or predicate)
- Purpose: relate rows from two tables according to some criteria
- Output: for each row r in R and each row s in S, output a row rs if r and s satisfy p
- Shorthand for $\sigma_p(R \times S)$

Join example

- Info about users, plus IDs of their groups

$$User \bowtie_{User.uid=Member.uid} Member$$

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	gid
123	gov
857	abc
857	gov
…	…

$\bowtie_{User.uid=Member.uid}$

uid	name	age	pop	uid	gid
123	Milhouse	10	0.2	123	gov
[redacted]					
857	Lisa	8	0.7	857	abc
857	Lisa	8	0.7	857	gov
…	…	…	…	…	…

Prefix a column reference with table name and "." to disambiguate identically named columns from different tables

Derived operator: natural join

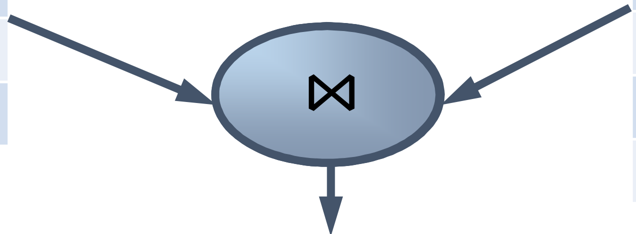
- Input: two tables R and S
- Notation: $R \bowtie S$
- Purpose: relate rows from two tables, and
  - Enforce equality between identically named columns
  - Eliminate one copy of identically named columns
- Shorthand for $\pi_L(R \bowtie_p S)$, where
  - p equates each pair of columns common to R and S
  - L is the union of column names from R and S (with duplicate columns removed)

Natural join example

$$User \bowtie Member = \pi_{?}(User \bowtie_{?} Member)$$
$$= \pi_{uid,name,age,pop,gid}\left(User \bowtie_{\substack{User.uid= \\ Member.uid}} Member\right)$$

uid	name	age	pop
123	Milhouse	10	0.2
857	Lisa	8	0.7
…	…	…	…

uid	gid
123	gov
857	abc
857	gov
…	…



uid	name	age	pop	[redacted]	gid
123	Milhouse	10	0.2	[redacted]	gov
[redacted]					
857	Lisa	8	0.7	[redacted]	abc
857	Lisa	8	0.7	[redacted]	gov
…	…	…	…	[redacted]	…

Union

- Input: two tables R and S
- Notation: $R \cup S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R and all rows in S (with duplicate rows removed)

Difference

- Input: two tables R and S
- Notation: $R - S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows in R that are not in S

Derived operator: intersection

- Input: two tables R and S
- Notation: $R \cap S$
  - R and S must have identical schema
- Output:
  - Has the same schema as R and S
  - Contains all rows that are in both R and S
- Shorthand for $R - (R - S)$
- Also equivalent to $S - (S - R)$
- And to $R \bowtie S$

Renaming

- Input: a table R and S
- Notation: $\rho_S\ R$, $\rho_{(A_1,A_2,\dots)}R$, or $\rho_{S(A_1,A_2,\dots)}R$
- Purpose: "rename" a table and/or its columns
- Output: a table with the same rows as R, but called differently
- Used to
  - Avoid confusion caused by identical column names
  - Create identical column names for natural joins
- As with all other relational operators, it doesn't modify the database
  - Think of the renamed table as a copy of the original

Renaming example

- IDs of users who belong to at least two groups

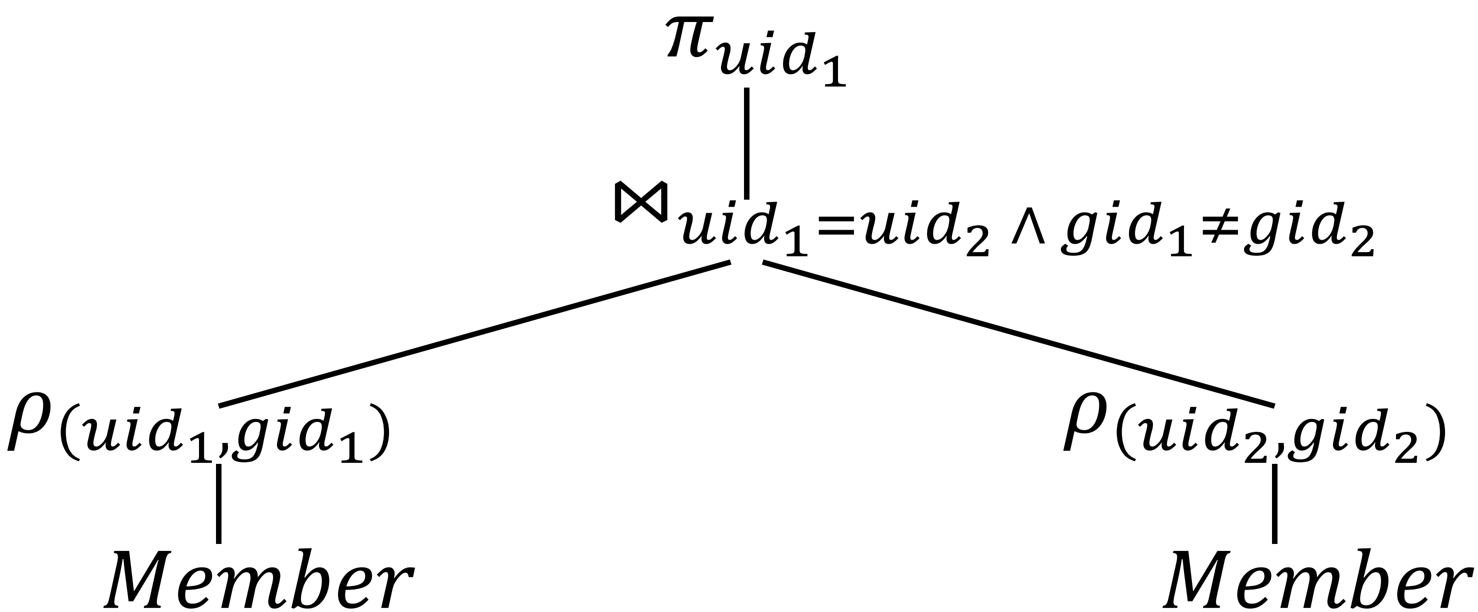
$$Member \bowtie_{?} Member$$

$$\pi_{uid}\left(Member \bowtie_{\substack{Member.uid=Member.uid \,\wedge \\ Member.gid\neq Member.gid}} Member\right)$$

WRONG!

$$\pi_{uid_1}\left(\begin{array}{c}\rho_{(uid_1,gid_1)} Member \\ \bowtie_{uid_1=uid_2 \,\wedge\, gid_1\neq gid_2} \\ \rho_{(uid_2,gid_2)} Member\end{array}\right)$$

Expression tree notation



Summary of core operators

- Selection: $\sigma_p R$
- Projection: $\pi_L R$
- Cross product: $R \times S$
- Union: $R \cup S$
- Difference: $R - S$
- Renaming: $\rho_{S(A_1, A_2, \dots)} R$
  - Does not really add "processing" power

Summary of derived operators

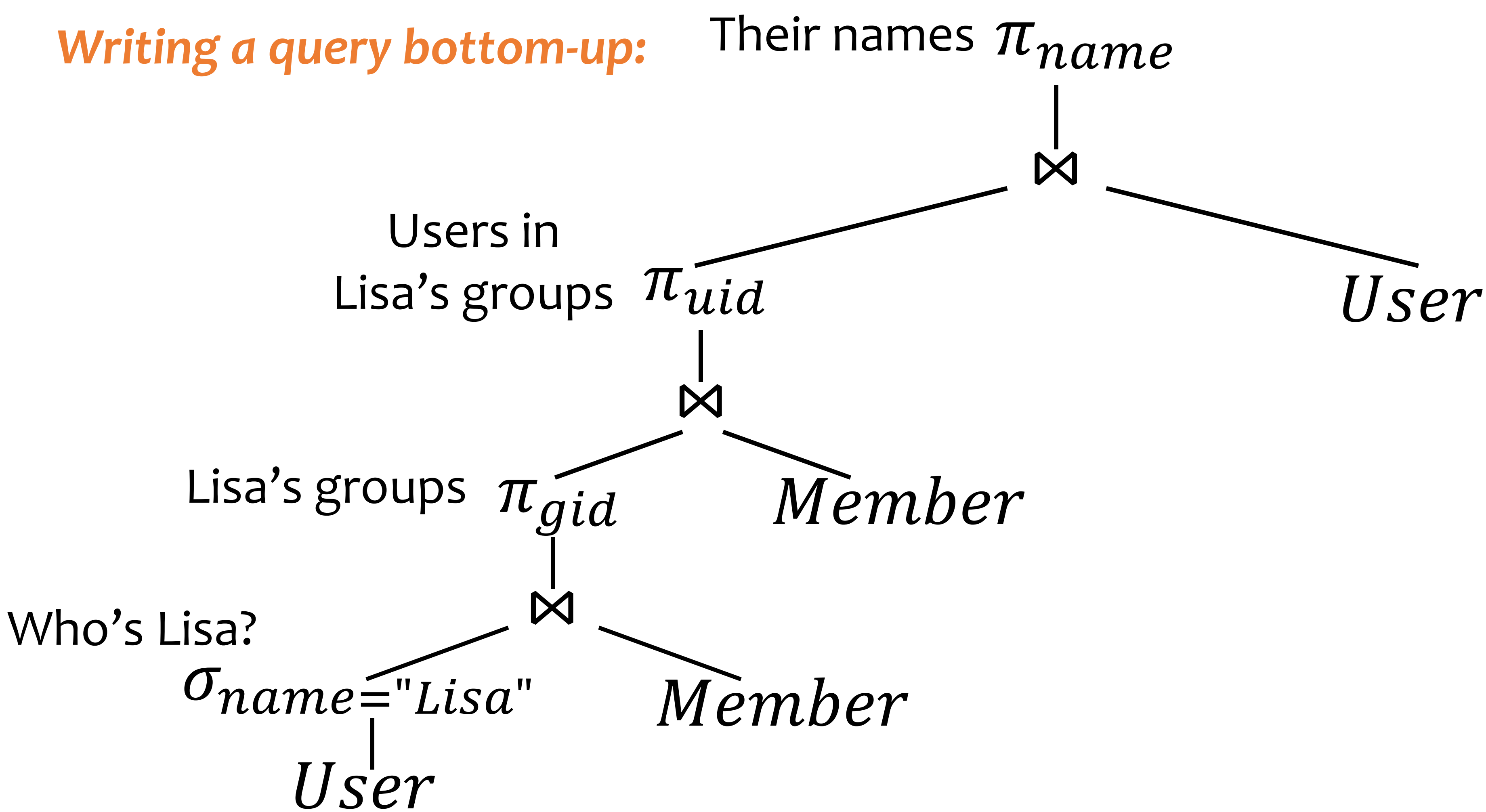
- Join: $R \bowtie_p S$
- Natural join: $R \bowtie S$
- Intersection: $R \cap S$

- Many more
  - Semijoin, anti-semijoin, quotient, …

An exercise

- Names of users in Lisa's groups

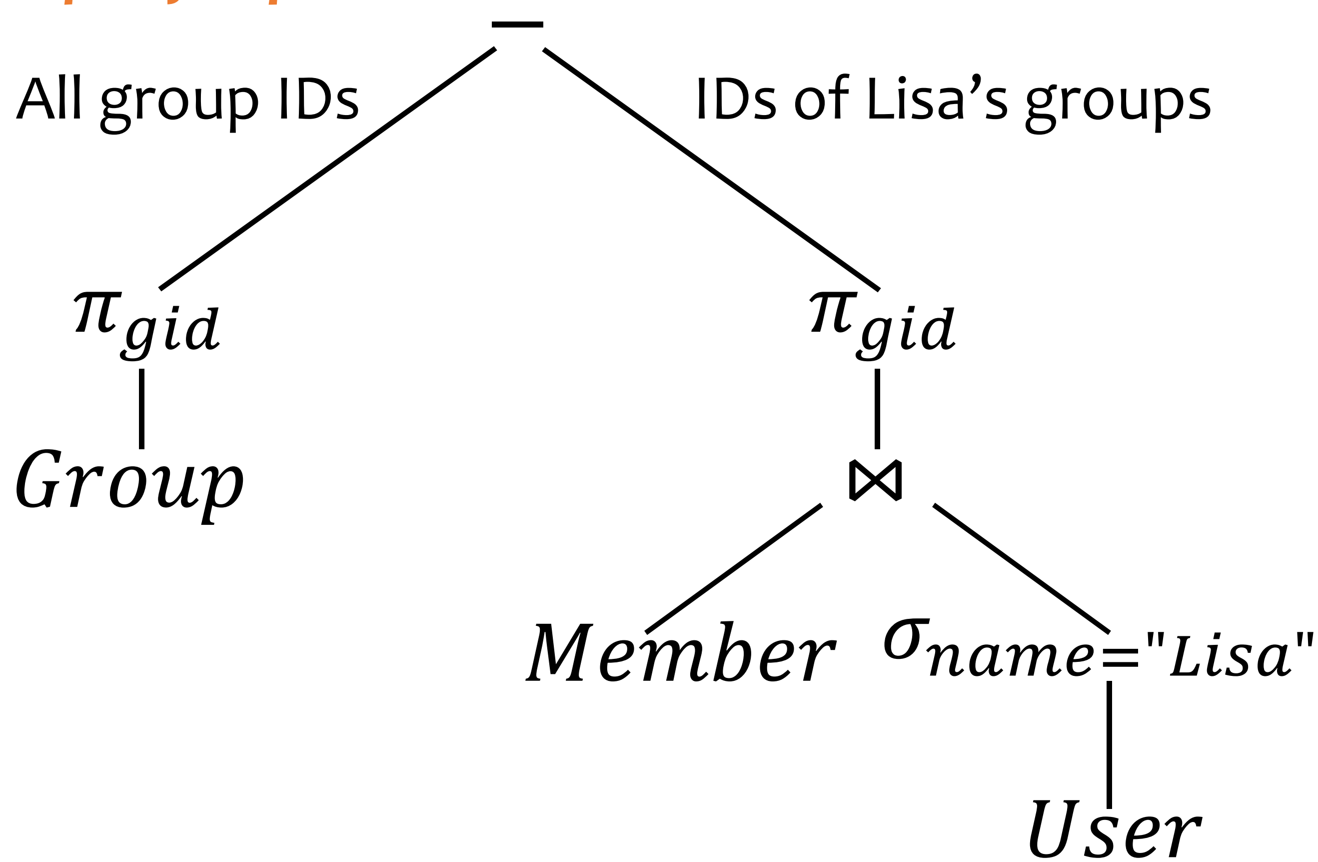
Writing a query bottom-up:



Another exercise

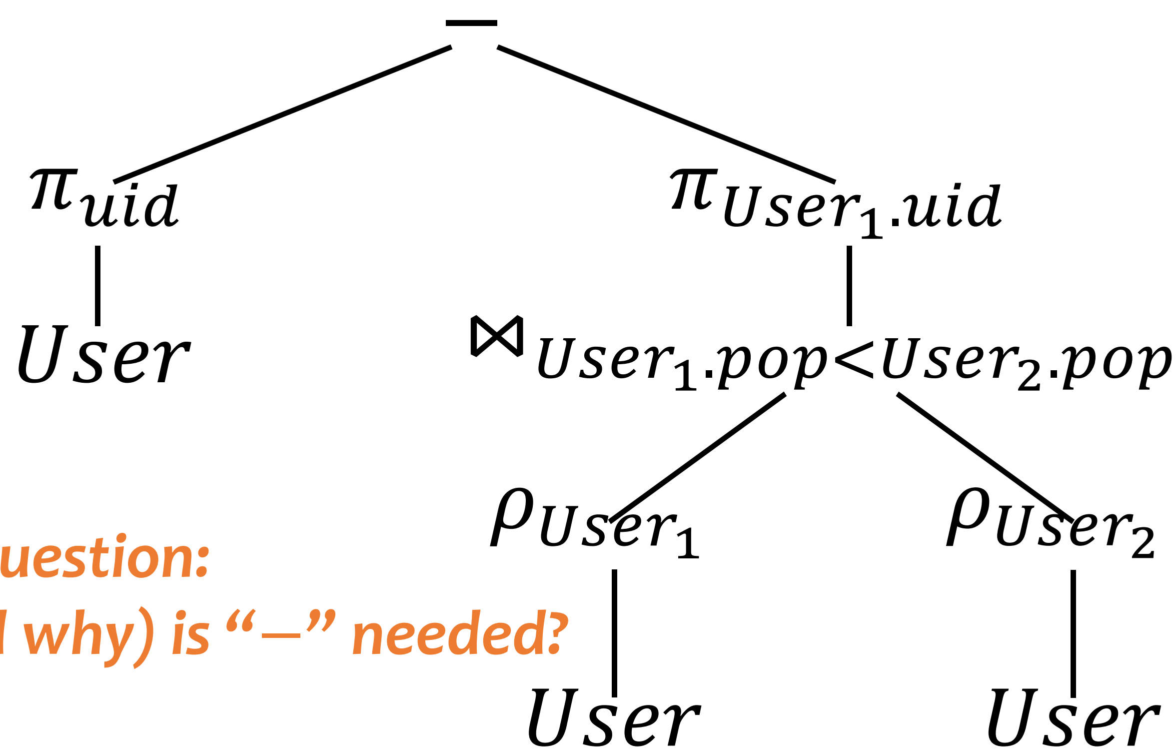
- IDs of groups that Lisa doesn't belong to

Writing a query top-down:



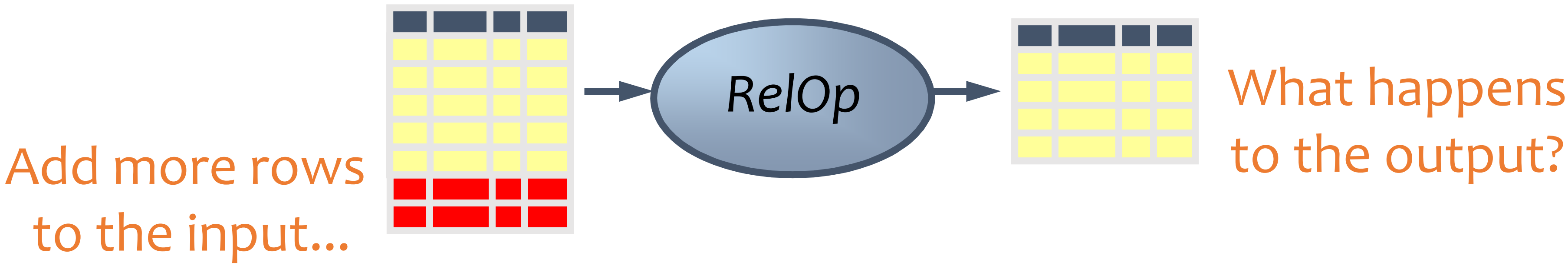
A trickier exercise

- Who are the most popular?
  - Who do NOT have the highest *pop* rating?
  - Whose *pop* is lower than somebody else's?



A deeper question:
When (and why) is "−" needed?

Monotone operators



- If some old output rows may need to be removed
  - Then the operator is non-monotone
- Otherwise the operator is monotone
  - That is, old output rows always remain "correct" when more rows are added to the input
- Formally, for a monotone operator op:
  $R \subseteq R'$ implies $op(R) \subseteq op(R')$ for any R, R'

Classification of relational operators

- Selection: $\sigma_p R$ — Monotone
- Projection: $\pi_L R$ — Monotone
- Cross product: $R \times S$ — Monotone
- Join: $R \bowtie_p S$ — Monotone
- Natural join: $R \bowtie S$ — Monotone
- Union: $R \cup S$ — Monotone
- Difference: $R - S$ — Monotone w.r.t. R; non-monotone w.r.t S
- Intersection: $R \cap S$ — Monotone

Why is "–" needed for "highest"?

- Composition of monotone operators produces a monotone query
  - Old output rows remain "correct" when more rows are added to the input
- Is the "highest" query monotone?
  - No!
  - Current highest *pop* is 0.9
  - Add another row with *pop* 0.91
  - Old answer is invalidated

☞So it must use difference!

Why do we need core operator X?

- Difference
  - The only non-monotone operator
- Cross product
  - The only operator that adds columns
- Union
  - The only operator that allows you to add rows?
  - A more rigorous argument?
- Selection? Projection?
  - Homework problem

Extensions to relational algebra

- Duplicate handling ("bag algebra")
- Grouping and aggregation
- "Extension" (or "extended projection") to allow new column values to be computed

☞All these will come up when we talk about SQL

☞But for now we will stick to standard relational algebra without these extensions

Why is r.a. a good query language?

- Simple
  - A small set of core operators
  - Semantics are easy to grasp
- Declarative?
  - Yes, compared with older languages like CODASYL
  - Though operators do look somewhat "procedural"
- Complete?
  - With respect to what?

Relational calculus

- $\{u.uid \mid u \in User \wedge$
  $\neg(\exists u' \in User: u.pop < u'.pop)\}$, or
- $\{u.uid \mid u \in User \wedge$
  $(\forall u' \in User: u.pop \geq u'.pop)\}$
- Relational algebra = "safe" relational calculus
  - Every query expressible as a safe relational calculus query is also expressible as a relational algebra query
  - And vice versa
- Example of an "unsafe" relational calculus query
  - $\{u.name \mid \neg(u \in User)\}$
  - Cannot evaluate it just by looking at the database

Turing machine

- A conceptual device that can execute any computer algorithm
- Approximates what general-purpose programming languages can do
  - E.g., Python, Java, C++, …

Alan Turing (1912-1954)

☞So how does relational algebra compare with a Turing machine?

http://en.wikipedia.org/wiki/File:Alan_Turing_photo.jpg

Limits of relational algebra

- Relational algebra has no recursion
  - Example: given relation *Friend*(*uid1*, *uid2*), who can Bart reach in his social network with any number of hops?
    - Writing this query in r.a. is impossible!
  - So r.a. is not as powerful as general-purpose languages
- But why not?
  - Optimization becomes undecidable

  ☞Simplicity is empowering
  - Besides, you can always implement it at the application level, and recursion is added to SQL nevertheless!